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The interpenetration of two clouds of rarefied plasma is involved in many problems of plasma physics, astrophysics, and geophysics. If the relative velocity of the plasma clouds is sufficiently large compared with the thermal velocities, the linear theory predicts instability with respect to the longitudinal electrostatic fluctuations [1]. The initial energy of oriented relative motion of the plasma clouds is converted, to some degree or other, into vibrational energy, to which the energy of the electric field already contributes significantly. In principle, this energy conversion may be so complete that the interpenetration of the plasma clouds takes the form of an inelastic collision. If the clouds have equal mass and one of them is at rest, then one-half the energy of oriented motion is dissipated as vibrational energy. In a rarefied plasma particle collisions play no part. Hence there is no reason to expect the dissipated energy fraction to be distributed among the plasma particles according to Maxwell's law, and the electric fields vanish. However, nonlinear interactions in the spectrum of the oscillations considerably accelerate the processes of randomization and "Maxwellization" of the vibrational motion of the plasma. The nonlinear problem of the interpenetration of plasma clouds can be studied only by numerical methods. A plasma model that permits such a numerical investigation of the problem is proposed below. A similar model has already been used by Buneman [2] and Dawson [3], and, for calculating the effect of the space charge in an accelerator, by Lomnev [4]. An approximate solution, based on the linear theory, is given by Parker [5], but in this paper the principal conclusions appear to have little justification. In the present paper some numerical results are presented. For the sake of simplicity the one-dimensional problem is considered, a direction being assigned to the relative velocity of the plasma clouds. In the actual calculations, protons were used as ions, i.e., $M/m = 1837$.

1. Discrete Plasma Model and Starting Equations

We shall write the initial one-dimensional kinetic equation of the α -component of the plasma ($\alpha = i, e$ are ions and electrons) in the form

$$\frac{\partial F_\alpha}{\partial t} + \frac{p}{m_\alpha} \frac{\partial F_\alpha}{\partial x} + e_\alpha E \frac{\partial F_\alpha}{\partial p} = 0 \quad (1.1)$$

where the random quantity

$$F_\alpha = \sum_i \delta [x - x_i^\alpha(t)] \delta [p - p_i^\alpha(t)] \quad (1.2)$$

is evaluated by summing with respect to all the elementary plasma particles the product of the two δ -functions. The necessary and sufficient condition for the function $F_\alpha(x, p, t)$ to satisfy Eq. (1.1) is the fulfillment of the equations of mechanics for any i -th particle

$$p_i^\alpha(t) = e_\alpha E, \quad x_i^\alpha(t) = \frac{1}{m_\alpha} p_i^\alpha(t). \quad (1.3)$$

Only the force due to the electric field E is considered in Eq. (1.1), although this limitation is not necessary.

From (1.1), or more precisely from its three-dimensional analog, all the well-known plasma kinetic equations can be obtained by introducing averaged distribution functions, as shown by Klimontovich [6]. Buneman [2] was the first to propose a finite system of equations (1.3) as a model and to use its numerical solution to study the nonlinear processes of electrostatic plasma oscillations.

In accordance with the foregoing, this is equivalent to solving the kinetic equation (1.1), which is also the starting equation for the kinetic equation of the plasma taking into account the effect of pair interactions. Consequently, the plasma model for a limited number of macroparticles, which corresponds to a finite number of equations (1.3), not only describes the motion of the plasma in a self-consistent electric field but also includes the effect of pair interactions. In the three-dimensional case for macroparticles with mass qm_e , charge qe , and velocity v_0 , the mean free path, defined as usual, is

$$l_0 \approx \frac{m_e^2 v_0^4}{ne^4 L} \frac{1}{q} \quad (1.4.1)$$

where L is the Coulomb logarithm, assumed to be considerably greater than unity, and n is the plasma electron density. In order to solve the problem of the effect of pair interactions in the plasma model considered below, it is

sufficient to consider only the case of macroelectrons in (1.4.1). In the one-dimensional problem, formula (1.4.1) is retained except that instead of the Coulomb logarithm the constant $4\pi^2$ is introduced. If v_0 is the velocity of the macroparticles — equal to the velocity of the plasma electrons — the mean free path of the macroparticles is found to be, in accordance with (1.4.1), less than the mean free path of the plasma electrons by a factor q . In other words, the efficiency of the pair interactions is q times greater. On the other hand, in this same model the characteristics of the collective plasma motions (wavelength, frequency, etc.) do not change. Hence, in the model the real relation between the effects of the collective and pair interactions is distorted. Consequently, the model will satisfactorily describe the nonlinear electrostatic plasma oscillations provided that the effect of pair interactions is small. We shall formulate the corresponding condition. The characteristic wavelength of the oscillations must be much less than the mean free path l_0 from (1.4.1), i. e., $v_0/\Omega_0 \ll l_0$. The ratio of these quantities, if we introduce the number of macroparticles per new unit length, $v_0/k\Omega_0$, is

$$\frac{v_0}{\Omega_0 l_0} = \frac{1}{4k^3 a^3} \ll 1, \quad a = \left(\frac{n}{q}\right)^{1/3} \frac{v_0}{k\Omega_0}. \quad (1.4.2)$$

In the problem considered below $a = l_0 n_0^* = 4$ and $k = 2^{-1}$ (see § 2), and the ratio obtained is equal to 32^{-1} . Obviously, such a criterion is too strict, since it does not allow for the self-consistent field, which considerably exceeds the electric field of a single macroparticle in this case. It follows naturally from (1.4.2) that the number of macroparticles per unit length must be large enough to prevent interactions of the macroparticles from playing a significant role.*

The laws of conservation of momentum and energy follow directly from Eq. (1.1). The momentum and kinetic energy of the particles are determined in terms of the function F_α

$$I(t) = \int_{-\infty}^{+\infty} p \sum_{\alpha} F_{\alpha}(x, p, t) dx dp, \quad W(t) = \int_{-\infty}^{+\infty} \frac{p^2}{2} \sum_{\alpha} \frac{F_{\alpha}(x, p, t)}{m_{\alpha}} dx dp. \quad (1.5)$$

If the electric field satisfies Maxwell's equations and vanishes at infinity, i. e., there is no external field, and the functions F_α are equal to zero when $|p| \rightarrow \infty$ (more precisely, $p^2 F_\alpha \rightarrow 0$ when $|p| \rightarrow \infty$), then the laws of conservation of momentum and energy hold:

$$\frac{\partial I}{\partial t} = 0, \quad \frac{\partial}{\partial t} \left(W + \frac{1}{8\pi} \int_{-\infty}^{+\infty} E^2 dx \right) = 0 \quad (1.6)$$

the electric field being linked with the particle density by the relation

$$\frac{\partial E}{\partial x} = 4\pi \sum_{\alpha} e_{\alpha} f_{\alpha}, \quad f_{\alpha} = \int_{-\infty}^{+\infty} F_{\alpha} dp. \quad (1.7)$$

The problem now consists in going over to the solution of a finite (computer-limited) number of equations (1.3) which nevertheless simulate the actual plasma. For this purpose, adjacent particles in phase space are combined to form built-up macroparticles, the number of which, in accordance with the foregoing [cf. (1.4.2)], should be quite large.

We shall introduce the concept of a macroparticle consisting of q elementary particles with the same velocity and coordinate. First, we shall assume that all macroparticles have the same value of q . For an electron-proton plasma we obtain from (1.3) and (1.7) the system of equations

$$m \frac{d^2 x_{ei}}{dt^2} = -eE, \quad M \frac{d^2 x_{pi}}{dt^2} = eE, \quad (1.8)$$

$$\frac{\partial E}{\partial x} = 4\pi e q (n_p - n_e), \quad (1.9)$$

where n_p and n_e are the densities of the proton and electron macroparticles, and i is the ordinal number of the electron or proton macroparticle. If Eq. (1.9) is integrated, then, since $E(-\infty) = 0$,

$$E(x, t) = 4\pi e q (N_p - N_e) \quad (1.10)$$

* Neglecting the effect of pair interactions in the plasma leads to a milder criterion

$$\frac{v_0}{\Omega_0 l_0 q} = \frac{1}{4a^3 k^3 q} \ll 1 \quad (a \sim n^{-1/3}).$$

where

$$N_\alpha(r, t) = \int_{-\infty}^x n_\alpha(x', t) dx' \quad (1.11)$$

denotes the number of macroparticles of the α -component of the plasma to the left of the point with coordinate x in a column with unit cross-sectional area.

In Eqs. (1.8) and (1.10) we shall go over to dimensionless units of time and length. Let

$$\tau = k\Omega_0 t, \quad \xi = \frac{k\Omega_0 x}{v_0}, \quad \Omega_0 = \left(\frac{4\pi e^2 q n_0}{m} \right)^{1/2}. \quad (1.12)$$

Here Ω_0 is the electron plasma frequency, n_0 is the characteristic macroparticle density, v_0 is the characteristic velocity, k is an arbitrary numerical coefficient. By means of Eqs. (1.12) we obtain from (1.8) and (1.10) in dimensionless variables the two groups of equations

$$\frac{d^2 \xi_{ei}}{d\tau^2} = -\frac{\Omega_0}{v_0 n_0 k} (N_p - N_e), \quad \frac{d^2 \xi_{pi}}{d\tau^2} = \frac{m}{M} \frac{\Omega_0}{v_0 n_0 k} (N_p - N_e) \quad (1.13)$$

In the one-dimensional problem it is natural to speak about macrolayers, which consolidate all the macroparticles with the same coordinate x . Essentially, this new concept does not change anything, except the dimensionality of the quantities n_0 and N_α . Consequently, we shall continue to use the name macroparticles for macrolayers. Now, however, we shall understand by n_0 only the characteristic linear density of the macroparticles.* If, finally, for each macroparticle we introduce the self-action and by ξ_i understand the coordinate of its center of mass, then the final form of Eqs. (1.13) will be:

$$\frac{d^2 \xi_{ei}}{d\tau^2} = -\frac{1}{b} \left[N_p(\xi_{ei}) - N_e(\xi_{ei}) - \frac{1}{2} \right], \quad (1.14)$$

$$\frac{d^2 \xi_{pi}}{d\tau^2} = \frac{m}{M} \frac{1}{b} \left[N_p(\xi_{pi}) + \frac{1}{2} - N_e(\xi_{pi}) \right]. \quad (1.15)$$

The quantities

$$b = k^2 n_0^*, \quad n_0^* = n_0 \frac{r_0}{k\Omega_0} \quad (1.16)$$

occur in Eqs. (1.14) and (1.15). Here n_0^* is the characteristic linear density of the macroparticles, referred to the unit of length introduced in Eq. (1.12).

The initial thermal spread at each point in space can be taken into account by dividing both the electron and the proton macroparticles into l smaller macroparticles with weight q_s for the s -th type (here $s = 1, \dots, l$). If we normalize q_s with the condition

$$q_1 + \dots + q_l = 1 \quad (1.17)$$

then the equations of motion for the i -th macroparticle of the s -th type will have the form:

$$\frac{d^2 \xi_{ei}}{d\tau^2} = \frac{q_{ei}}{2b} + \frac{1}{b} \left[\sum_{s=1}^l q_s N_e^s(\xi_{ei}) - \sum_{s=1}^l q_s N_p^s(\xi_{ei}) \right], \quad (1.18)$$

$$\frac{M}{m} \frac{d^2 \xi_{pi}}{d\tau^2} = \frac{q_{pi}}{2b} + \frac{1}{b} \left[\sum_{s=1}^l q_s N_p^s(\xi_{pi}) - \sum_{s=1}^l q_s N_e^s(\xi_{pi}) \right]. \quad (1.19)$$

The sums in Eq. (1.18) have the following meaning:

$$\sum_{s=1}^l q_s N_e^s(\xi_{ei}) \text{ — the sum of the weights of all the electron macroparticles,}$$

$$\sum_{s=1}^l q_s N_p^s(\xi_{ei}) \text{ — the sum of the weights of all the proton macroparticles to the left of the macroelectron with the coordinate } \xi_{ei}.$$

The sums in Eq. (1.19) have the same meaning, with the sole difference that summation of the weights of the particles is carried out with respect to all the particles located to the left of the macroproton with the coordinate ξ_{pi}

* Of course, in Ω_0 , as previously by n_0 , we must understand that the volume density of the macroparticles is intended.

Thus, we see the generality of the right sides of Eqs. (1.18) and (1.19). If we also assume that all the particles (irrespective of whether they are electrons or protons) are numbered in order of increase in their coordinates, then it is possible to combine Eqs. (1.18) and (1.19) as follows:

$$\lambda \frac{d^2 \xi_i}{d\tau^2} = \frac{\gamma_i}{2b} + \frac{1}{b} \sum_{\mu=1}^{i-1} \gamma_{\mu} \quad \text{for } (i = 1, 2, \dots, 2N). \quad (1.20)$$

Here $2N$ is the number of all the particles and γ_i is the weight of the i -th particle

$$\gamma_i = \begin{cases} q_i & \text{for an electron particle} \\ -q_i & \text{for a proton particle} \end{cases}$$

$$\lambda = \begin{cases} 1, & \text{if } \xi_i \text{ is the coordinate of an electron,} \\ -M/m, & \text{if } \xi_i \text{ is the coordinate of a proton.} \end{cases}$$

The laws of conservation of momentum and energy Eq. (1.6) have an important role in the solution of the problem and for a discrete plasma model take the form:

$$\frac{d}{d\tau} \left[\sum_{i=1}^{2N} \gamma_i \lambda \frac{d\xi_i}{d\tau} \right] = 0 \quad (1.21)$$

$$\frac{d}{d\tau} \left[\frac{1}{2} \sum_{i=1}^{2N} \gamma_i \lambda \left(\frac{d\xi_i}{d\tau} \right)^2 - \frac{1}{2b} \sum_{i=1}^{2N} \gamma_i \xi_i \left(\gamma_i + 2 \sum_{\mu=1}^{i-1} \gamma_{\mu} \right) \right] = 0. \quad (1.22)$$

Equations (1.21) and (1.22) follow immediately from (1.20). The dimensionless electrostatic energy

$$\varepsilon = \frac{1}{2b} \sum_{i=1}^{2N} \gamma_i \xi_i \left(\gamma_i + 2 \sum_{\mu=1}^{i-1} \gamma_{\mu} \right) \quad (1.23)$$

appears in the law of conservation of energy. It is easy to show that ε from (1.23) is equal to

$$\varepsilon = \frac{1}{mqv_0^2} \int_{-\infty}^{+\infty} \frac{E^2}{8\pi} dx, \quad (1.24)$$

E being given by Eq. (1.10).

2. Formulation of Problem Concerning the Collision of Plasma Clouds and Choice of Method of Calculation.

The idealized problem of the collision of two plasma clouds can be formulated in relation to the system of equations obtained at the end of §1. Above all, the idealization consists in the fact that the two clouds are not confined in directions perpendicular to the ξ axis. Further, in the given formulation the role of the magnetic field cannot be taken into account. Finally, the development of oscillations in oblique waves (the vector \mathbf{k} does not coincide in direction with the velocity v_0) is also disregarded. However, none of these limitations on the physical meaning of the problem is decisive.

Let us consider part of the real axis $-L < \xi < L$ and divide it into $2N$ equal segments. Suppose that at the instant $\tau = 0$ for $-L < \xi < 0$ (first cloud) there is a single macroelectron and a single macroproton in each of these segments. They are moving at an average velocity of -1 and have a small thermal spread described by the above-mentioned breakdown of macroparticles into smaller macroparticles. In a segment of length L/N there are l proton and l electron macroparticles. Their coordinates and order of arrangement in the segment are random, but they are concentrated in δ -neighborhood of the center of the segment. The second cloud is similarly distributed for $0 < \xi < L$, the particles being on the average at rest and having the same thermal spread. The particles thus disposed are numbered in order of increase in coordinates.

Random distribution of the particles in the segment is achieved by algebraic addition of the coordinate of the center of the segment and random numbers normalized so that the new coordinate does not leave the δ -neighborhood of the center.

The process of interpenetration commences at some subsequent instant ($\tau > 0$). Interaction with the development of longitudinal electrostatic oscillations begins with small random disturbances of electrical neutrality due to the spread of the macroparticles described. In other words, when $\tau = 0$, we have $\varepsilon = \varepsilon_0 \neq 0$, where ε_0 , in accordance with physical concepts, must not exceed the initial thermal energy of the electrons. The origin of the quantity

ϵ_0 , generally speaking, is connected with fluctuation processes in the plasma.

Thus, the important parameter L , the thickness of the plasma cloud, appears in the initial conditions of the problem, together with the unimportant parameters of the thermal spread of the macroparticles and the initial electrostatic energy ϵ_0 , provided they are sufficiently small. The parameter $b = k^2 N/L$, which appears in the equation, is expressed in terms of L . The given number of macroparticles N determines the accuracy of approximation to a continuous plasma, and is in fact determined by the computer capacity, but the parameter k is at our disposal.

Finally, in order to solve the problem it is necessary to consider a one-parameter family of variants. The dimensionless thickness of the plasma cloud L will serve as the characteristic parameter; L varies within such wide limits that at minimum L there are almost no interactions even between electron components of the plasma clouds, while at maximum L the interaction is considerable even between proton components. In the case of maximum L collision is inelastic. If the thermal spread is small, then its variation is not obligatory. It is given in the following manner in all the variants: a Maxwellian distribution is approximated by four macroparticles at each point, i.e., the number of types $l = 4$.

By making the proton and electron temperatures equal, we make the thermal velocity spread for the proton macroparticles less by a factor of $(M/m)^{1/2}$. The thermal spread of the electrons of both clouds is given in the table [q_s and $\xi_e(0)$].

cloud	$s=1$ $q_1=1/4$	$s=2$ $q_2=1/4$	$s=3$ $q_3=1/4$	$s=4$ $q_4=1/4$
$v_s = -1$ $v_s = 0$	-0.8 -0.2	-0.9 -0.1	-1.1 0.1	-1.2 0.2

For the purpose of ensuring the same accuracy of calculation a constant macroparticle density $N/L = 1$ was assigned in all the variants (different values of L). A value of $k = 2^{-1}$ was assumed. Then b is constant and equal to 4^{-1} .

The time interval considered in the calculations must exceed the effective interaction interval of the clouds, which, of course, is determined by the condition of their superposition in space:

$$\tau = 2L.$$

Thus, $\tau_k \geq 2L$. In order to determine the subsequent development of the oscillations it is desirable to prolong the calculations somewhat.

In order to obtain a numerical solution for Eqs. (1.20), we can either use the well-known numerical methods for solving ordinary differential equations or make an exact calculation, i.e., one that takes into account each intersection of the trajectories of any two neighboring particles.

The first method is very simple, but in this case it has an important disadvantage: the total energy of the system is not conserved. This is because a transition from Eqs. (1.20) to the law of conservation of total energy (1.22) is possible only if the intersections are accurately taken into account. In general, from (1.20) we get a somewhat different expression:

$$\frac{d}{d\tau} \left[\frac{1}{2} \sum_{i=1}^{2lN} \gamma_i \lambda \left(\frac{d\xi_i}{d\tau} \right)^2 - \frac{1}{2b} \sum_{i=1}^{2lN} \gamma_i \xi_i \left(\gamma_i + 2 \sum_{\mu=1}^{i-1} \gamma_\mu \right) \right] = - \frac{1}{2b} \sum_{i=1}^{2lN} 2\gamma_i \xi_i \frac{d}{d\tau} \sum_{\mu=1}^{i-1} \gamma_\mu. \quad (2.1)$$

On the right side of (2.1) there is a sum which obviously is zero if there are no intersections during the given interval and nonzero if intersections do occur.

In the numerical integration of (1.20) by existing difference methods, the difference analog of (2.1) will be valid, assuming stability and approximation in the usual sense (see [7]), for the numerical solution obtained.

We shall assume that in the interval $\Delta\tau$ there are no multiple particle intersections and consider two intersecting particles* with coordinates ξ' and ξ'' and weights γ' and γ'' . We shall also assume that in the interval $\Delta\tau$ there are no other intersections and that prior to the instant of intersections τ_0

$$\xi' < \xi'', \quad \xi' > \xi'' \quad \text{for } \tau > \tau_0.$$

Then the right side of Eq. (2.1), which is equal to zero for $\tau < \tau_0$, for $\tau > \tau_0$ is approximately:

$$- \frac{1}{2b} \sum_{i=1}^{2lN} 2\gamma_i \xi_i \frac{d}{d\tau} \sum_{\mu=1}^{i-1} \gamma_\mu \approx - \frac{\gamma' \gamma''}{b \Delta\tau} (\xi' - \xi'') \Big|_{\tau > \tau_0}. \quad (2.2)$$

* The possible simultaneous intersection of a particle with several others may be called multiple intersection. In this case, the basic argument from (2.2) remains valid, but the proof is more cumbersome.

Hence it follows that in the difference calculation the total energy of the system increases as a result of the intersection of unlike charges and decreases upon the intersection of like particles.

It may be expected that in this problem there will be considerably more intersections of unlike than of like charges. In this case there will be an increase in the total energy of the system, as also observed in the calculations (see §3). Since the number of intersections increases sharply with time, a serious breakdown of the law of conservation of total energy is to be expected in the later stages of the calculations.

Note that in the law of conservation of total momentum of the system, which also serves as a criterion of the accuracy of the calculations, there is no differentiation of discontinuous quantities. In calculating all the variants this law was fulfilled to eight places.

The exact method also has a serious disadvantage: in the simple realization, i.e., when the time step is equal to the interval between the two nearest intersections, the calculation interval for a variant increases to such an extent that integration of Eqs. (1.20) by this method has no sense.

With a view to a sensible compromise, the exact method was combined with the difference method for Eq. (1.20). For a given $\Delta\tau$, from known values of $(d\xi/d\tau)^n$ and ξ^n we found $(d\xi/d\tau)^{n+1}$ and ξ^{n+1} by the difference method. We then checked for particle intersections in the interval $\Delta\tau$. If there were intersections, then the values of $(d\xi/d\tau)^{n+1}$ for these particles were recalculated, taking into account the exact time of intersection. It was found possible to take accurately into account two successive intersections of any particle. However, the idea of taking into account three or more intersections of the same particle during the interval $\Delta\tau$ had to be abandoned, since the corresponding logic was extremely complex and the calculating time for a single step increases sharply. The time step $\Delta\tau$ was chosen as $(1/100)\pi$ in all the variants, and the maximum number of particles possible in the program was 1960.

3. Discussion of Results

A series of calculations was made with $L = 10, 30, 50, 70,$ and 100 . The results were expressed in the form of several integral and average quantities as a function of time. As these quantities we selected the momentum and kinetic energy of the electrons, respectively:

$$I_e = \sum_i q_i \dot{\xi}_{ei}, \quad W_e = \frac{1}{2} \sum_i q_i (\dot{\xi}_{ei})^2,$$

the average initial electron velocities of the moving and stationary clouds, v_{p1} and v_{p2} , respectively:

$$v_{ej} = \frac{\sum q_i \dot{\xi}_{ei}}{\sum q_i} \quad (i \in j),$$

the electrostatic energy ϵ , determined from formula (1.23), and, finally, the proton velocities v_{p1} and v_{p2} , with the same significance as the analogous electron velocities. In addition, the total momentum and total energy of the entire system were calculated; these must be constant, in accordance with the laws of conservation. The latter served for checking the accuracy of the numerical calculations. Data concerning the local distribution of the quantities were not processed in this series of calculations.

The principal result was a demonstration of the existence of intense collective interaction between the plasma clouds from the very beginning of their interpenetration. This agrees with the linear theory, but it should be emphasized that the limits of applicability of the latter to the problem considered are very narrow.

We shall give a qualitative picture of the process, based on the linear theory [5], and at the same time point out its limitations. In the region of interpenetration of the plasma clouds there develops, above all, a collective interaction between the two electron components. The densities of the two clouds are identical, and therefore their interaction can "randomize" the relative motion of the electrons in a few periods of the plasma oscillations. The electron distribution function at the end of this stage of interaction can not be determined on the basis of the linear theory. Further development of the collective processes is associated with interaction between the collectivized electron cloud and the two proton components, which have almost monochromatic distribution functions. This interaction leads to a transfer of energy from the protons to the electrons. The transferred energy is many times greater than the energy of random motion of the electrons at the end of the electron-electron interaction stage. This energy transfer process again significantly changes the electron distribution function and likewise can not be described within the framework of the linear theory.

As soon as the electrons have acquired an energy of the order of the energy of oriented motion of the protons, the conditions are created for the development of direct interaction between the proton components of the clouds — the excitation of low-frequency ionic oscillations. It should be mentioned, however, that at this stage the starting state of the proton components is substantially different from their initial state, because the energy of oriented motion of the protons of the moving cloud has already been to a considerable extent converted into the energy of electron-ion

oscillations. Consequently, the stage of development of ionic oscillations again cannot be quantitatively described by the linear theory. The residue of the energy of oriented motion is converted into ionic oscillations, and this concludes the process of plasma cloud interaction of interest to us. The subsequent state of the system may possibly be associated with prolonged damping of the excited oscillations, as in [3]. If the dimensions of the clouds L are not sufficiently large, then the above process of development of the interactions terminates when the clouds again become separated in space.

We have not raised the problem of a detailed comparison of the numerical calculations with the results of the linear theory, as Dawson, for example, did in [3], because, firstly, this is relatively ineffective owing to the narrow range of applicability of the linear theory, and, secondly, very time-consuming owing to the three-dimensional nature of the problem, in contrast to [3]. However, the general features of the interaction process, derived from the linear theory (for example, the different stages of interaction) are useful and are essentially confirmed by the numerical calculations.

Let us pass immediately to a discussion of the results obtained. Figure 1 shows curves of time ($\tau_K \approx 80$) vs. W_e , I_e , v_{e1} , v_{e2} and ε for the variant $L = 30$. Figure 1 includes W_e curves for slightly different initial electrostatic energies ε_0 and completely different initial particle spreads (see § 2). It should be remarked that in general outline the picture of the process is the same. The kinetic energy of the electrons exceeds the initial kinetic energy of the oriented and thermal motion by a factor of four to five, and the electrostatic energy increases by a factor of 30-40 compared with ε_0 . There is an increase in the energy $W_e + \varepsilon$ at the expense of a reduction in proton energy. On the average, the electron velocities v_{e1} and v_{e2} tend towards a value of -0.5 , which corresponds to electron collectivization. The period of the oscillations is $T \approx 2.3$ for $\tau \approx 30-50$ and $T \approx 3.1$ for $\tau \approx 50-70$ (I_e curve). On the other hand, the period of the plasma oscillations in an electron plasma of doubled density:

$$\tau^* = k\Omega_0 \frac{2\pi}{\sqrt{2}\Omega_0} \approx 2.24,$$

while for normal density $\tau^* \approx 3.18$. According to the linear theory this is precisely the fundamental period of the electron-ion oscillations [8]. The general orderliness of electron motion, characteristic of the electron-ion interaction stage and steadily increasing with increase in L , indicates that the oscillations are quite homogenous throughout the cloud. In the initial interaction stage, up to $\tau \approx 10-20$, on the other hand, the curves are all very irregular. Here, according to the linear theory, there is probably a so-called spatial build-up of the oscillations. It should be noted that there is no significant polarization of the clouds during the process of oscillation. In fact, if the characteristic velocity of the electrons is ~ 1 , then during a half-period ~ 1.5 electrons are displaced relative to the position of the protons by ~ 1.5 , which is considerably less than the cloud thickness $L = 30$.

Finally, the curves have a particularly interesting feature at $\tau \geq 70$, i.e., when the plasma clouds have already separated in space. There is a well-expressed tendency for W_e and ε to increase slightly. A similar picture is also observed in the variant $L = 10$. Simultaneously with the increase in W_e and ε there is an intensification of the proton velocity oscillations. Obviously, the strong oscillations of the electron component, associated with cloud interaction, condition in each cloud the further dissipation of the energy of oriented motion of the protons into electron-ion oscillations.

As mentioned above, the law of conservation of energy serves as a check on the accuracy of the numerical calculations. It has been assumed that all the results obtained are meaningful, so long as the inequality

$$W_e + \varepsilon \gg \Delta W \quad (\Delta W = W(\tau) - W_0) \quad (3.1)$$

is fulfilled. Here ΔW characterizes the infringement of the law of conservation of energy, and W_0 is the initial total energy of the system. Inequality (3.1) is a much stronger requirement than the trivial condition

$$W \gg \Delta W \quad (3.2)$$

because the fraction of electron energy summed with the electrostatic energy is very small compared with the proton energy. For example, in the variant in question, $L = 30$, we have $W_e + \varepsilon \approx 0.004 W$ toward the end of the process. Given condition (3.1), we can state that the increase in the energy of the electrons in the electron-ion oscillations occurs at the expense of the proton energy (we recall, incidentally, that in the calculation process $\Delta W > 0$). Condition (3.1) was checked in all the variants calculated. The condition was violated at $\tau \approx 80$ for variants $L = 70$ and $L = 100$ alike, when the kinetic energy of the electrons exceeded their initial energy by approximately 50 times. In the variant $L = 100$, calculated up to $\tau \approx 180$, condition (3.2) was also violated toward the end of the calculations, but the electron energy at this time was of the order of W_0 . However, the last result must evidently be regarded as purely qualitative. Figure 2 shows $W(\tau)$ in relative units. The graph starts at a moment ($\tau \approx 60$) when the energy W already differs from $W_0 = 739$ in the third digit. The smooth increase in $W(\tau)$, compared with the other variables is worth noting. This type of increase may be expected on the basis of the explanation given in § 2. The curves of v_{e2} and v_{p2} for the variant $L = 100$ (Fig. 3) are included to give a qualitative illustration of the process of ordered oscillations. Initially,

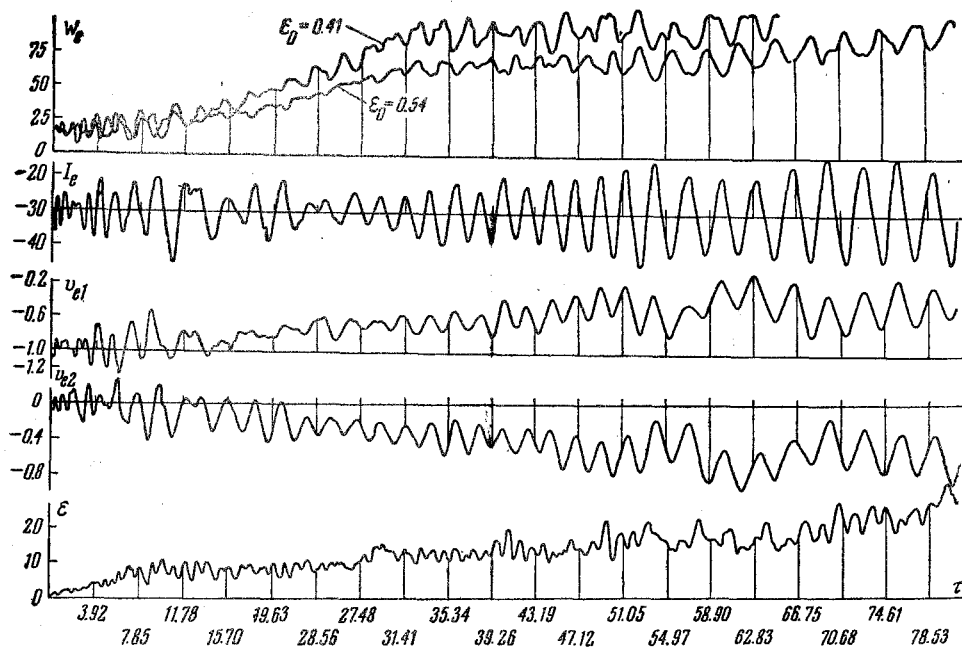


Fig. 1.

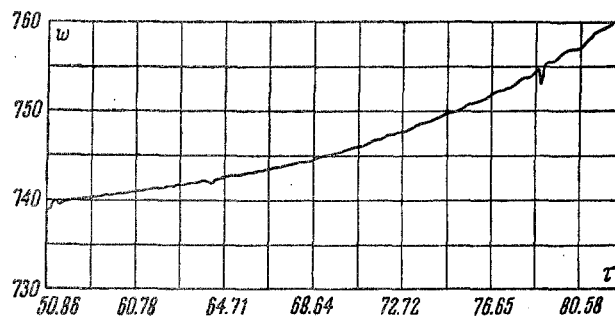


Fig. 2.

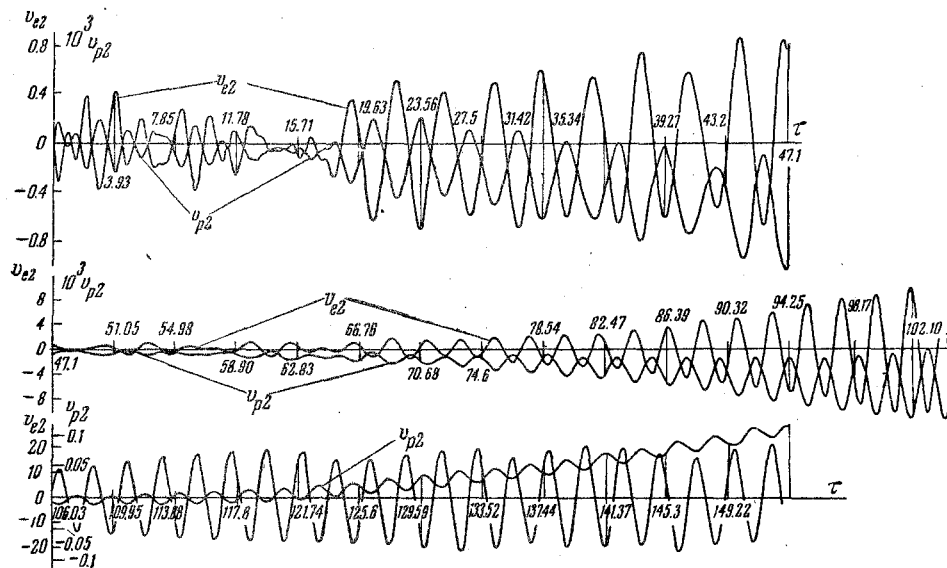


Fig. 3.

the v_{p2} scale is 10^8 times larger than the scale of v_{e2} . It can be seen from Fig. 3 that the oscillations of the proton velocity are displaced in phase by π relative to the oscillations in electron velocity. In addition, the curve v_{p2} experiences low-frequency oscillations with a period of several tens of units. Note that the ionic plasma period for normal density is $T \approx 130$. It is quite possible for the numerical calculations to detect the development of ionic oscillations, especially as the variation of v_{p1} is correlated with the variation of v_{p2} . At $\tau \approx 180$, the proton velocity v_{p2} reaches 0.4, i. e., the velocity of vibrational motion is comparable with the velocity of oriented motion.

In order to obtain well-founded quantitative results for the $L = 70$ and $L = 100$ variants, it is desirable to make new calculations free of the shortcoming mentioned above – violation of the law of conservation of energy.

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